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CSCI 4446 Problem Set 1

**Logistic maps**

xn+1 = Rxn(1 − xn) where R >= 0

When R is small (i.e. less than 1) the population will eventually go extinct, as the growth rate is only a fraction of the population. As R is between the range of (1, 3) the population will grow to some nonzero steady state, essentially its “limit”. For a higher growth rate, R, say between [3, 3.6), the population will begin to oscillate over its prior steady state; that is transitioning between max and mins of the population in a periodic fashion, actually referred to as period-doubling as R increases. The next set range of [3.6, 4] represents the chaotic regime, where the system will behave chaotically. If a rate of 4 is exceeded the system collapses.

The graph below demonstrates a period 2 cycle (beginning when R = 3) that no longer converges to a precise stable level, but transitions between a min and max range along that previous steady state around X(n) = .65. This is the first occurrence of a bifurcate, where the possible population values split into two discrete paths.

The next figure demonstrates an 8 period cycle, happening just after the growth rate passes 3.5 where the population values bifurcate yet again, this time into eight new paths, or eight population values.

The final two Poincare plots, or phase diagrams, are among the range of the chaotic regime, where R ranges [3.6, 4] and behavior is classified as chaotic. If the system were not chaotic at this point the diagrams would show the attractor points, 1 point if it’s stable or multiple points equivalent to the period-doubling cycle. These plots show chaos as they form parabolic curves demonstrating some sense of attractors but without ever converging to some fixed point or controlled oscillation as earlier period-doubling cycles did, and rather jumping around to nonrepeating population values.

 When R = 2.5 the population value always goes back to 0.6 despite given any initial condition in range (0, 1). You could say this is a linear dynamical system as the system is not sensitively dependent on the initial conditions, but rather stable in convergence to a future value (of .6).